Fock States of Flavor Neutrinos are Unphysical

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Abstract

It is shown that it is possible to construct an infinity of Fock spaces of flavor neutrinos depending on arbitrary unphysical mass parameters, in agreement with the theory of Blasone and Vitiello in the version proposed by Fujii, Habe and Yabuki. However, we show by *reductio ad absurdum* that these flavor neutrino Fock spaces are clever mathematical constructs without physical relevance, because the hypothesis that neutrinos produced or detected in charged-current weak interaction processes are described by flavor neutrino Fock states implies that measurable quantities depend on the arbitrary unphysical flavor neutrino mass parameters.

1 Introduction

Neutrino oscillations [1–3] is one of the main fields of contemporary experimental and theoretical research in high-energy physics. The main reason is that neutrino oscillations is a consequence of neutrino mixing (see Refs. [4–12] and the the recent review by B. Kayser in Ref. [13]), which consists in a mismatch between flavor and mass: the left-handed flavor neutrino fields $\nu_{\alpha L}$, with $\alpha = e, \mu, \tau$, are unitary linear combinations of the massive neutrino fields ν_{kL},

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \qquad (\alpha = e, \mu, \tau) \, , \tag{1.1}$$

where U is the mixing matrix. Since neutrinos are massless in the Standard Model, neutrino oscillations represents an open window on the physics beyond the Standard Model (see Refs. [14–17]) The theory of neutrino oscillations has been discussed in many papers (see Ref. [18]) and reviewed in Refs. [4, 5, 9, 19–23].

The standard derivation of the neutrino oscillation probability follows from the description of neutrinos produced or detected in charged-current weak interaction processes through the flavor neutrino states

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \qquad (\alpha = e, \mu, \tau) , \qquad (1.2)$$

where $|\nu_k\rangle$ is the state of a neutrino with mass m_k, which belongs to the Fock space of the quantized massive neutrino field ν_k.

It must be noted that the flavor state (1.2) is *not* a quantum of the flavor field ν_α [24]. Indeed, one can easily check that the flavor state (1.2) is not annihilated by the flavor field ν_α if the neutrino masses are taken into account.

In Ref. [24] it was argued that it is impossible to construct a Fock space of flavor states. In the proof of this statement it was implicitly assumed that would-be creation (destruction) operators of flavor states can be linear combinations of creation (destruction) operators of massive states only, excluding a contribution from destruction (creation) operators of massive states. As explained in Section 2, this assumption, although physically reasonable, is inconsistent with the theory. It follows that it is possible to construct a Fock space of flavor states, as it was first noticed by Blasone and Vitiello (BV) [25] in 1995 and later discussed in several papers by BV with collaborators [26–28], by Fujii, Habe and Yabuki (FHY) [29, 30], by Blasone et al. [31], by Ji and Mishchenko [32], and other more mathematically oriented authors [33]. Actually, as shown by FHY [29], there is an infinity of flavor Fock spaces depending on arbitrary unphysical mass parameters.

It is then necessary to determine if the flavor Fock states can describe neutrinos produced or detected in charged-current weak interaction processes. As discussed in Section 3 our conclusion is negative, showing that the flavor Fock spaces are clever mathematical constructs without physical relevance. Let us emphasize that this fact precludes the description of neutrinos in oscillation experiments through the flavor Fock states, because these neutrinos must be produced and detected in weak interaction processes.

The plane of the paper is as follows. In Section 2 we review the argument presented in Ref. [24] against a Fock space of flavor states, we show its inconsistency and we explain how an infinity of Fock spaces of flavor states can be constructed, obtaining the BV and FHY results through a different way. In Section 3 we show that the flavor Fock spaces are unphysical and in Section 4 we summarize our conclusions.

2 Fock space of flavor fields

There is neutrino mixing if the mass matrix is not diagonal in the basis of the flavor neutrino fields $\nu_\alpha(x)$, where $\alpha = e, \mu, \tau$ is the flavor index. If we consider, for simplicity, the mixing of three Dirac neutrinos, the flavor neutrino fields are related to the massive

neutrino fields $\nu_k(x)$, where $k = 1, 2, 3$ is the mass index, by the mixing relation[1]

$$\nu_\alpha(x) = \sum_k U_{\alpha k}\, \nu_k(x)\,, \tag{2.1}$$

where U is the unitary 3×3 mixing matrix.

The quantized massive neutrino fields $\nu_k(x)$ obey the canonical equal-time anticommutation relations

$$\{\nu_{k\xi}(t, \vec{x})\,,\, \nu^\dagger_{j\eta}(t, \vec{y})\} = \delta(\vec{x} - \vec{y})\, \delta_{kj}\, \delta_{\xi\eta}\,, \tag{2.2}$$

where ξ and η are Dirac indices $(\xi, \eta = 1, \ldots, 4)$, and

$$\{\nu_{k\xi}(x)\,,\, \nu_{j\eta}(y)\} = \{\nu^\dagger_{k\xi}(x)\,,\, \nu^\dagger_{j\eta}(y)\} = 0\,. \tag{2.3}$$

Since the quantized massive neutrino fields must satisfy the free Dirac equation, they can be Fourier expanded as

$$\nu_k(x) = \int \frac{\mathrm{d}\vec{p}}{(2\pi)^{3/2}} \sum_{h=\pm 1} \left[a_{\nu_k}(\vec{p}, h)\, u_{\nu_k}(\vec{p}, h)\, e^{-iE_{\nu_k}t + i\vec{p}\vec{x}} + b^\dagger_{\nu_k}(\vec{p}, h)\, v_{\nu_k}(\vec{p}, h)\, e^{iE_{\nu_k}t - i\vec{p}\vec{x}} \right]\,, \tag{2.4}$$

where $E_{\nu_k} = \sqrt{\vec{p}^2 + m^2_{\nu_k}}$, h is the helicity, $u_{\nu_k}(\vec{p}, h)$ and $v_{\nu_k}(\vec{p}, h)$ are the usual four-component spinors in momentum space such that

$$(\not{p} - m_{\nu_k})\, u_{\nu_k}(\vec{p}, h) = 0\,, \qquad (\not{p} + m_{\nu_k})\, v_{\nu_k}(\vec{p}, h) = 0\,, \tag{2.5}$$

for which we use the BV normalization [25]

$$u^\dagger_{\nu_k}(\vec{p}, h)\, u_{\nu_k}(\vec{p}, h') = v^\dagger_{\nu_k}(\vec{p}, h)\, v_{\nu_k}(\vec{p}, h') = \delta_{hh'}\,. \tag{2.6}$$

The following orthogonality and completeness relations are useful:

$$u^\dagger_{\nu_k}(\vec{p}, h)\, v_{\nu_k}(-\vec{p}, h') = 0\,, \tag{2.7}$$

$$\sum_h \left(u_{\nu_k}(\vec{p}, h)\, u^\dagger_{\nu_k}(\vec{p}, h) + v_{\nu_k}(-\vec{p}, h)\, v^\dagger_{\nu_k}(-\vec{p}, h) \right) = 1\,. \tag{2.8}$$

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[1] More precisely, there are two mixing relations for the left and right handed fields in the basis in which the mass matrix of the charged lepton fields is diagonal:

$$\nu_{\alpha L}(x) = \sum_k U_{\alpha k}\, \nu_{kL}(x)\,, \qquad \nu_{\alpha R}(x) = \sum_k V_{\alpha k}\, \nu_{kR}(x)\,,$$

with the unitary matrices U and V such that the mass matrix M is diagonalized by the biunitary transformation $V^\dagger M U = M_{\mathrm{diag}}$ (see Ref. [5]). However, since the right-handed fields $\nu_{\alpha R}(x)$ do not participate to weak interactions, we can define appropriate right-handed flavor fields

$$\nu'_{\alpha R}(x) = \sum_\beta \left(U V^\dagger \right)_{\alpha\beta} \nu_{\beta R}(x) = \sum_k U_{\alpha k}\, \nu_{kR}(x)\,,$$

such that the flavor fields $\nu_\alpha(x) = \nu_{\alpha L}(x) + \nu'_{\alpha R}(x)$ satisfy the mixing relations in Eq. (2.1).

Using the orthonormality relations (2.6) and (2.7), one can find that

$$a_{\nu_k}(\vec{p}, h) = \int \frac{\mathrm{d}\vec{x}}{(2\pi)^{3/2}} \, e^{iE_{\nu_k}t - i\vec{p}\vec{x}} \, u_{\nu_k}^\dagger(\vec{p}, h) \, \nu_k(x) \,, \tag{2.9}$$

$$b_{\nu_k}(\vec{p}, h) = \int \frac{\mathrm{d}\vec{x}}{(2\pi)^{3/2}} \, \nu_k^\dagger(x) \, v_{\nu_k}(\vec{p}, h) \, e^{iE_{\nu_k}t - i\vec{p}\vec{x}} \,. \tag{2.10}$$

The canonical anticommutation relations (2.2) and (2.3) for the massive neutrino fields imply that

$$\{a_{\nu_k}(\vec{p}, h) \,, a_{\nu_j}^\dagger(\vec{p}', h')\} = \{b_{\nu_k}(\vec{p}, h) \,, b_{\nu_j}^\dagger(\vec{p}', h')\} = \delta(\vec{p} - \vec{p}') \, \delta_{hh'} \, \delta_{kj} \,, \tag{2.11}$$

and all the other anticommutation relations vanish. Since these are the canonical anti-commutation relations for fermionic ladder operators, the operators $a_{\nu_k}^\dagger(\vec{p}, h)$ and $b_{\nu_k}^\dagger(\vec{p}, h)$ can be interpreted, respectively, as the one-particle and one-antiparticle creation opera-tors which allow to construct the Fock space of massive neutrino states starting from the vacuum ground state $|0\rangle$.

Let us now consider the flavor fields $\nu_\alpha(x)$. In order to generate a Fock space of flavor states, the Fourier expansion of the flavor fields must be written as

$$\nu_\alpha(x) = \int \frac{\mathrm{d}\vec{p}}{(2\pi)^{3/2}} \sum_{h=\pm 1} \left[a_{\nu_\alpha}(\vec{p}, h) \, u_{\nu_\alpha}(\vec{p}, h) \, e^{-iE_{\nu_\alpha}t + i\vec{p}\vec{x}} + b_{\nu_\alpha}^\dagger(\vec{p}, h) \, v_{\nu_\alpha}(\vec{p}, h) \, e^{iE_{\nu_\alpha}t - i\vec{p}\vec{x}} \right] \,, \tag{2.12}$$

where $E_{\nu_\alpha} = \sqrt{\vec{p}^2 + \widetilde{m}_{\nu_\alpha}^2}$ with arbitrary mass parameters $\widetilde{m}_{\nu_\alpha}$, and the spinors $u_{\nu_\alpha}(\vec{p}, h)$ and $v_{\nu_\alpha}(\vec{p}, h)$ are assumed to satisfy equations analogous to the ones in Eq. (2.5) [29]:

$$(\not{p} - \widetilde{m}_{\nu_\alpha}) \, u_{\nu_\alpha}(\vec{p}, h) = 0 \,, \qquad (\not{p} + \widetilde{m}_{\nu_\alpha}) \, v_{\nu_\alpha}(\vec{p}, h) = 0 \,. \tag{2.13}$$

Hence, the spinors $u_{\nu_\alpha}(\vec{p}, h)$ and $v_{\nu_\alpha}(\vec{p}, h)$ satisfy orthonormality and completeness rela-tions analogous to those in Eqs. (2.6)–(2.8):

$$u_{\nu_\alpha}^\dagger(\vec{p}, h) \, u_{\nu_\alpha}(\vec{p}, h') = v_{\nu_\alpha}^\dagger(\vec{p}, h) \, v_{\nu_\alpha}(\vec{p}, h') = \delta_{hh'} \,, \tag{2.14}$$

$$u_{\nu_\alpha}^\dagger(\vec{p}, h) \, v_{\nu_\alpha}(-\vec{p}, h') = 0 \,, \tag{2.15}$$

$$\sum_h \left(u_{\nu_\alpha}(\vec{p}, h) \, u_{\nu_\alpha}^\dagger(\vec{p}, h) + v_{\nu_\alpha}(-\vec{p}, h) \, v_{\nu_\alpha}^\dagger(-\vec{p}, h) \right) = 1 \,. \tag{2.16}$$

Using Eq. (2.4), the mixing relation (2.1) allows to write the flavor fields as

$$\nu_\alpha(x) = \int \frac{\mathrm{d}\vec{p}}{(2\pi)^{3/2}} \sum_{h=\pm 1} \sum_k U_{\alpha k} \left[a_{\nu_k}(\vec{p}, h) \, u_{\nu_k}(\vec{p}, h) \, e^{-iE_{\nu_k}t + i\vec{p}\vec{x}} + b_{\nu_k}^\dagger(\vec{p}, h) \, v_{\nu_k}(\vec{p}, h) \, e^{iE_{\nu_k}t - i\vec{p}\vec{x}} \right] \,. \tag{2.17}$$

Confronting with Eq. (2.12) and assuming that the would-be destruction (creation) oper-ators of flavor states are linear combinations of destruction (creation) operators of massive states only, for the would-be destruction operators of flavor neutrino states $a_{\nu_\alpha}(\vec{p}, h)$ we have

$$a_{\nu_\alpha}(\vec{p}, h) \, u_{\nu_\alpha}(\vec{p}, h) \, e^{-iE_{\nu_\alpha}t} = \sum_k U_{\alpha k} \, a_{\nu_k}(\vec{p}, h) \, u_{\nu_k}(\vec{p}, h) \, e^{-iE_{\nu_k}t} \,. \tag{2.18}$$

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Using the orthonormality relation (2.14) we obtain

$$a_{\nu_\alpha}(\vec{p}, h) = \sum_k U_{\alpha k}\, a_{\nu_k}(\vec{p}, h)\left(u^\dagger_{\nu_\alpha}(\vec{p}, h)\, u_{\nu_k}(\vec{p}, h)\right)e^{i\left(E_{\nu_\alpha}-E_{\nu_k}\right)t}, \qquad (2.19)$$

With the help of Eq. (2.11) one can calculate the anticommutation relation

$$\{a_{\nu_\alpha}(\vec{p}, h)\,,\, a^\dagger_{\nu_\beta}(\vec{p}', h')\} = \delta(\vec{p} - \vec{p}')\,\delta_{hh'}\, e^{i\left(E_{\nu_\alpha}-E_{\nu_\beta}\right)t}$$

$$\times\, u^\dagger_{\nu_\alpha}(\vec{p}, h)\left(\sum_k U_{\alpha k}\, U^*_{\beta k}\, u_{\nu_k}(\vec{p}, h)\, u^\dagger_{\nu_k}(\vec{p}, h)\right) u_{\nu_\beta}(\vec{p}, h)\,, \quad (2.20)$$

which is not proportional to $\delta_{\alpha\beta}$ because of the 4×4 matrix coefficients $u_{\nu_k}(\vec{p}, h)\, u^\dagger_{\nu_k}(\vec{p}, h)$ that prevent the operativeness of the unitarity relation $\sum_k U_{\alpha k}\, U^*_{\beta k} = \delta_{\alpha\beta}$. A similar derivation applies to the operators $b_{\nu_\alpha}(\vec{p}, h)$.

From these considerations one can see that the operators $a_{\nu_\alpha}(\vec{p}, h)$ and $b_{\nu_\alpha}(\vec{p}, h)$ calculated in this way do not have the properties of fermionic ladder operators. From similar considerations, in Ref. [24] it was concluded that a Fock space of flavor states do not exist. Let us emphasize again that this conclusion follows from the assumption that the would-be destruction (creation) operators of flavor states are linear combinations of destruction (creation) operators of massive states only. This is equivalent to assume that the vacuum of the Fock space of flavor states is the same as the vacuum of the Fock space of massive states, because the vacuum of the Fock space of massive states is obviously annihilated by the operators $a_{\nu_\alpha}(\vec{p}, h)$ in Eq. (2.19) (and by the $b_{\nu_\alpha}(\vec{p}, h)$ defined in an analogous way). We think that this is a necessary requirement for a physical interpretation of Fock space of flavor states, because there is only one vacuum in the real world.

However, the condition that the would-be destruction (creation) operators of flavor states are linear combinations of destruction (creation) operators of massive states only is in contradiction with the Fourier expansion (2.12) of $\nu_\alpha(x)$ and the orthonormality relations of the spinors $u_{\nu_\alpha}(\vec{p}, h)$ and $v_{\nu_\alpha}(\vec{p}, h)$ in Eqs. (2.14)–(2.16), which imply that the operators $a_{\nu_\alpha}(\vec{p}, h)$ and $b_{\nu_\alpha}(\vec{p}, h)$ are given by relations analogous to those in Eqs. (2.9) and (2.10):

$$a_{\nu_\alpha}(\vec{p}, h) = \int \frac{d\vec{x}}{(2\pi)^{3/2}}\, e^{iE_{\nu_\alpha}t - i\vec{p}\vec{x}}\, u^\dagger_{\nu_\alpha}(\vec{p}, h)\, \nu_\alpha(x)\,, \qquad (2.21)$$

$$b_{\nu_\alpha}(\vec{p}, h) = \int \frac{d\vec{x}}{(2\pi)^{3/2}}\, \nu^\dagger_\alpha(x)\, v_{\nu_\alpha}(\vec{p}, h)\, e^{iE_{\nu_\alpha}t - i\vec{p}\vec{x}}\,. \qquad (2.22)$$

Using the mixing relation (2.1) and the Fourier expansion (2.4) of the massive neutrino fields, we obtain

$$a_{\nu_\alpha}(\vec{p}, h) = e^{iE_{\nu_\alpha}t}\sum_k U_{\alpha k}\left[a_{\nu_k}(\vec{p}, h)\left(u^\dagger_{\nu_\alpha}(\vec{p}, h)\, u_{\nu_k}(\vec{p}, h)\right)e^{-iE_{\nu_k}t}\right.$$

$$\left. + b^\dagger_{\nu_k}(-\vec{p}, h)\left(u^\dagger_{\nu_\alpha}(\vec{p}, h)\, v_{\nu_k}(-\vec{p}, h)\right)e^{iE_{\nu_k}t}\right]\,, \qquad (2.23)$$

$$b_{\nu_\alpha}(\vec{p}, h) = e^{iE_{\nu_\alpha}t}\sum_k U^*_{\alpha k}\left[a^\dagger_{\nu_k}(-\vec{p}, h)\left(u^\dagger_{\nu_k}(-\vec{p}, h)\, v_{\nu_\alpha}(\vec{p}, h)\right)e^{iE_{\nu_k}t}\right.$$

$$+ \, b_{\nu_k}(\vec{p}, h) \Big(v^\dagger_{\nu_k}(\vec{p}, h) \, v_{\nu_\alpha}(\vec{p}, h) \Big) e^{-iE_{\nu_k} t} \Bigg] \, . \tag{2.24}$$

These relations are identical to those obtained by FHY [29,30] (see also Ref. [27]) through a generalization of the BV formalism[2] [25]. The operators $a_{\nu_\alpha}(\vec{p}, h)$ and $b_{\nu_\alpha}(\vec{p}, h)$ satisfy the canonical anticommutation relations

$$\{a_{\nu_\alpha}(\vec{p}, h)\,,\, a^\dagger_{\nu_\beta}(\vec{p}', h')\} = \{b_{\nu_\alpha}(\vec{p}, h)\,,\, b^\dagger_{\nu_\beta}(\vec{p}', h')\} = \delta(\vec{p} - \vec{p}')\, \delta_{hh'}\, \delta_{\alpha\beta} \, , \tag{2.25}$$

and all the other anticommutation relations vanish. Therefore, the argument presented in Ref. [24] against a Fock space of flavor states is inconsistent and, as pointed out by BV [25], the operators $a^\dagger_{\nu_\alpha}(\vec{p}, h)$ and $b^\dagger_{\nu_\alpha}(\vec{p}, h)$ can be interpreted, respectively, as the one-particle and one-antiparticle creation operators which allow to construct a Fock space of flavor neutrino states starting from a vacuum ground state. However, such vacuum ground state is different from the vacuum ground state of massive neutrinos, that we have denoted by $|0\rangle$, as one can immediately see from the fact that the operators $a_{\nu_\alpha}(\vec{p}, h)$ and $b_{\nu_\alpha}(\vec{p}, h)$ in Eqs. (2.23) and (2.24) do not annihilate $|0\rangle$:

$$a_{\nu_\alpha}(\vec{p}, h)\, |0\rangle = e^{iE_{\nu_\alpha} t} \sum_k U_{\alpha k} \Big(u^\dagger_{\nu_\alpha}(\vec{p}, h)\, v_{\nu_k}(-\vec{p}, h) \Big) e^{iE_{\nu_k} t}\, |\bar{\nu}_k(-\vec{p}, h)\rangle \, , \tag{2.26}$$

$$b_{\nu_\alpha}(\vec{p}, h)\, |0\rangle = e^{iE_{\nu_\alpha} t} \sum_k U^*_{\alpha k} \Big(u^\dagger_{\nu_k}(-\vec{p}, h)\, v_{\nu_\alpha}(\vec{p}, h) \Big) e^{iE_{\nu_k} t}\, |\nu_k(-\vec{p}, h)\rangle \, . \tag{2.27}$$

Therefore, the vacuum ground state of the flavor neutrino Fock space is different from the vacuum ground state of the massive neutrino Fock space [25]. Actually, there is an infinity of Fock spaces of flavor neutrinos depending on the values of the arbitrary parameters $\widetilde{m}_{\nu_\alpha}$ [29].

Let us denote with $|0_{\{\widetilde{m}\}}\rangle$ the vacuum ground state of the flavor neutrino Fock space corresponding to a set of values of the parameters $\widetilde{m}_{\nu_\alpha}$. In principle we should add a suffix $\{\widetilde{m}\}$ also to the operators $a_{\nu_\alpha}(\vec{p}, h)$ and $b_{\nu_\alpha}(\vec{p}, h)$ in Eqs. (2.23) and (2.24), but we refrain from complicating the notation in such way, being understood that the operators are assumed to act on the corresponding vacuum state with the same values of the parameters $\widetilde{m}_{\nu_\alpha}$.

Having proved the mathematical possibility to construct Fock spaces of flavor neutrinos, it is necessary to investigate if these Fock spaces and their associated vacuums have any physical relevance. In Section 3 we will see that the hypothesis that real flavor neutrinos produced and detected in charged-current weak interaction processes are described by flavor Fock states leads to the absurd result that the arbitrary mass parameters $\widetilde{m}_{\nu_\alpha}$ are measurable. Hence, the only Fock space which describes reality is the massive neutrino Fock space and its vacuum ground state is the physical vacuum.

This fact may be also clear from the above derivation, in which we started with the quantization of the massive neutrino fields, which are the fundamental quantities, and we defined arbitrarily the Fourier expansion of the flavor fields in Eq. (2.12), through the arbitrary mass parameters $\widetilde{m}_{\nu_\alpha}$ and the arbitrary relations (2.13). Instead the masses of the massive neutrino fields and the relations (2.5) are not arbitrary, because they are determined by the Dirac Lagrangian, which implies free Dirac equations for the massive

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[2]BV assumed that $\widetilde{m}_{\nu_e} = m_{\nu_1}$, $\widetilde{m}_{\nu_\mu} = m_{\nu_2}$, $\widetilde{m}_{\nu_\tau} = m_{\nu_3}$.

neutrino fields. On the other hand, the flavor neutrino fields do not satisfy any sort of free Dirac equation, because neutrino mixing implies that the equations of the flavor neutrino fields are coupled by the off-diagonal mass terms in the flavor basis. Indeed, using Eqs. (2.13), (2.23) and (2.24) one can directly check that the flavor fields $\nu_\alpha(x)$ in Eq. (2.12) do not satisfy a free Dirac equation with mass $\widetilde{m}_{\nu_\alpha}$, because the operators $a_{\nu_\alpha}(\vec{p}, h)$ and $b_{\nu_\alpha}(\vec{p}, h)$ are time-dependent. Therefore, the definition of the spinors $u_{\nu_\alpha}(\vec{p}, h)$ and $v_{\nu_\alpha}(\vec{p}, h)$ through Eqs. (2.13) is completely arbitrary and the mass parameters $\widetilde{m}_{\nu_\alpha}$ are unphysical. Indeed, it has been emphasized by FHY [29] that the mass parameters $\widetilde{m}_{\nu_\alpha}$ should disappear in all measurable quantities. Since the Fourier expansion of the flavor fields in Eq. (2.12) is an arbitrary mathematical construct, we are not surprised by the fact that the corresponding Fock space of flavor neutrinos has no physical relevance.

3   Measurable Quantities

In Ref. [25] BV define the flavor one-neutrino state as

$$|\nu_\alpha(\vec{p}, h)\rangle = a^\dagger_{\nu_\alpha}(\vec{p}, h) |0\rangle , \tag{3.1}$$

whereas in Refs. [26–28] they adopt the definition

$$|\nu_\alpha(\vec{p}, h)\rangle = a^\dagger_{\nu_\alpha}(\vec{p}, h) |0_{\{\widetilde{m}\}}\rangle , \tag{3.2}$$

whose motivations are explained in Ref. [26]. It seems to us that it is obvious that the definition (3.2) is the correct one from the point of view of someone which believes that the Fock space of flavor states describes reality, because the states in Eq. (3.2) belong to such Fock space, whereas the states in Eq. (3.1) are time-dependent superpositions of states belonging to the Fock space of massive neutrinos, if $a_{\nu_\alpha}(\vec{p}, h)$ is interpreted according to Eq. (2.23).

In this Section we show that the interpretation of the definition (3.2) as a physical state describing a flavor neutrino produced or detected in a charged-current weak interaction process leads to the absurd result that the unphysical arbitrary parameters $\widetilde{m}_{\nu_\alpha}$ are measurable. This does not mean that the definition (3.1) is any better, as we will see in the following.

Let us consider the simplest case of the pion decay process

$$\pi^+ \to \mu^+ + \nu_\mu . \tag{3.3}$$

If the flavor one-neutrino states are real, the outgoing muon neutrino in Eq. (3.3) is described by the state

$$|\nu_\mu(\vec{p}, h)\rangle = a^\dagger_\mu(\vec{p}, h) |0_{\{\widetilde{m}\}}\rangle . \tag{3.4}$$

The amplitude of the decay is given by

$$\mathcal{A} = \langle \mu^+(\vec{p}_\mu, h_\mu), \nu_\mu(\vec{p}, h)| - i \int \mathrm{d}^4 x \, \mathcal{H}_\mathrm{I}(x) |\pi^+(\vec{p}_\pi), 0_{\{\widetilde{m}\}}\rangle , \tag{3.5}$$

where we have written explicitly the vacuum flavor state just to make clear that it is assumed to correspond to the physical vacuum. The effective interaction Hamiltonian $\mathcal{H}_\mathrm{I}(x)$ is given by

$$\mathcal{H}_\mathrm{I}(x) = \frac{G_\mathrm{F}}{\sqrt{2}} \, \overline{\nu_\mu}(x) \, \gamma^\rho \, (1 - \gamma_5) \, \mu(x) \, J_\rho(x) \,, \tag{3.6}$$

where G_F is the Fermi constant and $J_\rho(x)$ is the hadronic weak current, whose matrix element is given by

$$\langle 0 | J_\rho(x) | \pi^+(\vec{p}_\pi) \rangle = i \, \vec{p}_{\pi\rho} \, f_\pi \, \cos \vartheta_\mathrm{C} \, e^{-i p_\pi x} \,, \tag{3.7}$$

where f_π is the pion decay constant and ϑ_C is the Cabibbo angle (see, for example, Ref. [34]). Using Eqs. (2.12), (2.25) and (3.4) we obtain

$$\mathcal{A} = 2\pi \frac{G_\mathrm{F}}{\sqrt{2}} \, \vec{p}_{\pi\rho} \, f_\pi \, \cos \vartheta_\mathrm{C} \, \delta^4(p_\pi - p_\mu - p) \, \overline{u_{\nu_\mu}}(\vec{p}, h) \, \gamma^\rho \, (1 - \gamma_5) \, v_\mu(\vec{p}_\mu, h_\mu) \,. \tag{3.8}$$

It is clear that if this expression were correct the arbitrary unphysical mass parameter $\widetilde{m}_{\nu_\mu}$ would be a measurable quantity, because the energy of the muon neutrino is $E_{\nu_\mu} = \sqrt{\vec{p} + \widetilde{m}_{\nu_\mu}^2}$. Since the unphysical mass parameter $\widetilde{m}_{\nu_\mu}$ enters in the energy-conservation delta function and in the spinor $u_{\nu_\mu}(\vec{p}, h)$, it determines the measurable four-momentum of the muon through energy-momentum conservation and the measurable decay rate of the pion. Hence, we conclude that the state (3.4) is unphysical.

Considering other charged-current weak interaction processes one can rule out the physical relevance of the states (3.2) for all flavors α.

The definition (3.1), whatever its meaning, does not lead to anything better. In this case the amplitude of the pion decay (3.3) is given by

$$\mathcal{A} = \langle \mu^+(\vec{p}_\mu, h_\mu), \nu_\mu(\vec{p}, h) | - i \int \mathrm{d}^4 x \, \mathcal{H}_\mathrm{I}(x) | \pi^+(\vec{p}_\pi), 0 \rangle \,, \tag{3.9}$$

where we have written explicitly the vacuum state of the massive neutrino Fock space in order to make clear that it is assumed to correspond to the physical vacuum, and

$$|\nu_\mu(\vec{p}, h)\rangle = a_{\nu_\mu}^\dagger(\vec{p}, h) |0\rangle \,. \tag{3.10}$$

Using Eqs. (2.11), (2.17) and (2.23), for the matrix element of the neutrino field we obtain

$$\langle \nu_\mu(\vec{p}, h) | \overline{\nu_\mu}(x) | 0 \rangle = \frac{1}{(2\pi)^{3/2}} \, e^{i E_{\nu_\mu} t - i \vec{p} \vec{x}} \sum_k |U_{\mu k}|^2 \left(u_{\nu_\mu}^\dagger(\vec{p}, h) \, u_{\nu_k}(\vec{p}, h) \right) \overline{u_{\nu_k}}(\vec{p}, h) \,, \tag{3.11}$$

which implies that again both energy-momentum conservation and the pion decay rate depend on the unphysical mass $\widetilde{m}_{\nu_\mu}$.

Summarizing, we have shown that both the definitions (3.1) and (3.2) adopted by BV, FHY and others cannot correspond to a physical flavor neutrino state because they would imply that the arbitrary unphysical parameters $\widetilde{m}_{\nu_\alpha}$ are measurable[3]. Since the introduction of these unphysical parameters is necessary for the construction of a Fock space

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[3]Let us notice that also the arbitrary BV assumption $\widetilde{m}_{\nu_e} = m_{\nu_1}$, $\widetilde{m}_{\nu_\mu} = m_{\nu_2}$, $\widetilde{m}_{\nu_\tau} = m_{\nu_3}$ does not lead to acceptable results. For example, it would imply that the pion decay process (3.3) depends only on the neutrino mass m_{ν_2} if the definition (3.2) is adopted. On the other hand, using the definition (3.1) one obtains that energy-momentum conservation depends only on m_{ν_2}, although all the neutrino masses contribute in a complicated way to the decay rate.

of flavor neutrinos, we conclude that such Fock spaces are only mathematical constructs, without physical relevance.

Let us emphasize that the unacceptable results obtained in this Section are an unavoidable consequence of the hypothesis that the flavor Fock space is real, which means that flavor neutrinos are described by flavor Fock states. In this case it is not allowed to use the flavor Fock states for some calculations (for example neutrino oscillations) and the massive Fock states for other calculations (for example pion decay), all of which involve neutrinos created or detected in charged-current weak interactions[4]. The obvious reason is that the flavor Fock states, if real, are just the states which describe the neutrinos produced and detected in any charged-current weak interaction process, including those operating in neutrino oscillation experiments. In the calculation of these processes the flavor neutrino Fock states would have the same relevance as the Fock states of all other particles.

The correct way to calculate decay rates (as well as other processes) taking into account neutrino masses and mixing has been discussed in Refs. [35–37]. It is based on the fact that the massive neutrinos have definite kinematical properties and constitute the possible orthogonal asymptotic states of the decay. In other words, each decay in a massive neutrino constitutes a possible decay channel and the total decay probability is the sum of the decay probabilities in the different massive neutrinos ν_k weighted by the squared absolute value of the element of the mixing matrix that weights the contribution of ν_k to the charged-current weak interaction Hamiltonian. The description of neutrinos produced or detected in charged-current weak interaction processes through the standard flavor neutrino states (1.2) leads to the same result [38]. Hence, the standard flavor neutrino states (1.2) can be used to describe in a consistent framework neutrino interactions and oscillations in neutrino oscillation experiments.

4   Conclusions

We have shown that the argument presented in Ref. [24] against the existence of a Fock space of flavor neutrinos is inconsistent. Hence, we agree with BV, FHY and others [25–33] that it is possible to construct a Fock space of flavor neutrinos. However, there is an infinity of such Fock spaces of flavor neutrinos depending on the values of arbitrary unphysical mass parameters [29]. We have shown that the hypothesis that the flavor Fock states describe real flavor neutrinos produced or detected in weak interaction charged-current processes leads to the absurd consequence that the arbitrary unphysical mass parameters are measurable quantities. In particular, the flavor Fock states are inadequate for the description of flavor neutrinos in oscillations experiments, because these flavor neutrinos are produced and detected through weak interaction charged-current processes. Therefore, we conclude that the Fock spaces of flavor neutrinos are ingenious mathematical constructs without physical relevance.

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[4]Since we have shown that the flavor Fock space is unphysical, there is no need to discuss neutral-current weak processes.

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